

On the Explosion Mechanism of SNe Type Ia

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Abstract

In this article we discuss the first simulations of two- and three-dimensional Type Ia supernovae with an improved hydrodynamics code. After describing the various enhancements, the obtained results are compared to those of earlier code versions, observational data and the findings of other researchers in this field.

Key words: supernovae: general, physical data and processes: hydrodynamics, turbulence, nuclear reactions, nucleosynthesis, abundances, methods: numerical
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1 Introduction

The most generally accepted scenario for the explosion of a Type Ia supernova consists of a thermonuclear reaction front propagating through a white dwarf of Chandrasekhar mass, which is subsequently disrupted. Very early on it was discovered that this combustion process cannot start out as a supersonic detonation wave, but has to burn at subsonic speeds at least during the early stages, in order to explain the abundances observed in SN Ia remnants (see, e.g., ?).

This fact complicates the numerical modeling of such a process mainly for two reasons:

- The buoyancy of the hot ashes near the stellar center leads to the development of hydrodynamic instabilities on many scales, which have a significant influence on the explosion dynamics, mainly by increasing the total flame surface and therefore the energy generation rate. Only a small part of the

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relevant scales can be resolved, so that a large region of scale space must be described by numerical turbulence models.

- Since the resulting turbulent flow pattern inside the exploding white dwarf is inherently three-dimensional, reliable quantitative results can only be obtained by resource-intensive 3D numerical calculations. Two-dimensional simulations, which imply axial symmetry conditions, may exhibit a different turbulent energy and velocity cascade and also underestimate the total burning surface. Consequently, they can only provide qualitative results and serve for validation of the numerical models.

In the following we shortly describe the numerical techniques employed in our SN Ia code and discuss the results of 2D and 3D calculations and their relation to observational data.

2 Numerical methods

The ensemble of numerical techniques used for the presented simulations is based on those described by ?); however, several physical and numerical corrections and improvements were implemented in the meantime:

- The original minimalistic combustion scheme (one-step reaction from carbon/oxygen to nickel) was extended considerably. In the new version the composition of the ashes depends on the density of the fuel, i.e. a NSE-like mixture of nickel and α -particles is synthesized at high densities and ^{24}Mg (representing intermediate-mass elements) is produced at lower densities. Additionally, the composition in the Ni/ α material tracks the NSE equilibrium during the expansion of the star, leading to delayed energy release.
- An incorrect numerical factor was found and corrected in the calculation of the source terms of the two-dimensional sub-grid model for the flame propagation speed. This mistake did not have a significant effect on the results published earlier (see, e.g., ?), but becomes noticeable if resolution studies are performed (see the next section).
- All numerical models were extended to allow three-dimensional simulations in Cartesian coordinates. Since 3D calculations are much more resource-intensive than 2D runs with comparable resolution, a parallelization of the complete numerical code was also necessary in order to run it on massively parallel computers.

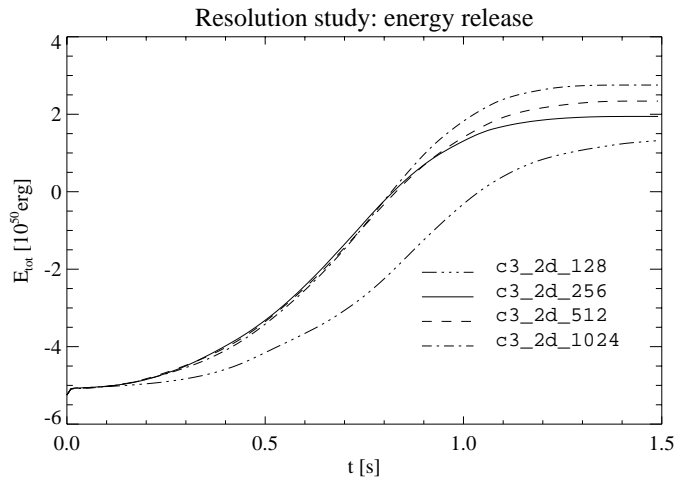


Fig. 1. Time evolution of the total energy for identical initial conditions at different resolutions. While model c3_2d_128 is clearly under-resolved, the other simulations agree very well, at least in the early and intermediate stages.

3 SN calculations

Different series of simulations were performed to check the numerical reliability of the employed models and to compare two- and three-dimensional explosions.

3.1 Resolution study

A crucial test for the validity of the models for the unresolved scales (in this case the flame and subgrid models) is to check the dependence of integral quantities, like the total energy release of the explosion, on the numerical grid resolution. Ideally, there should be no such dependence, indicating that all effects on unresolved scales are accurately modeled.

Figure 1 shows the energy evolution of a centrally ignited white dwarf. The only difference between the simulations is the central grid resolution, which ranges from $2 \cdot 10^6 \text{cm}$ (model c3_2d_128) down to $2.5 \cdot 10^5 \text{cm}$ (model c3_2d_1024). Model c3_2d_128 is obviously under-resolved, but the results of the other calculations are in good agreement, with exception of the last stages, where the flame enters strongly non-uniform regions of the grid.

So far, this kind of parameter study could only be performed in two dimensions, because of the prohibitive cost of very highly resolved 3D simulations. Nevertheless the results suggest that a resolution of $\approx 10^6 \text{cm}$ should yield acceptable accuracy also in three dimensions.

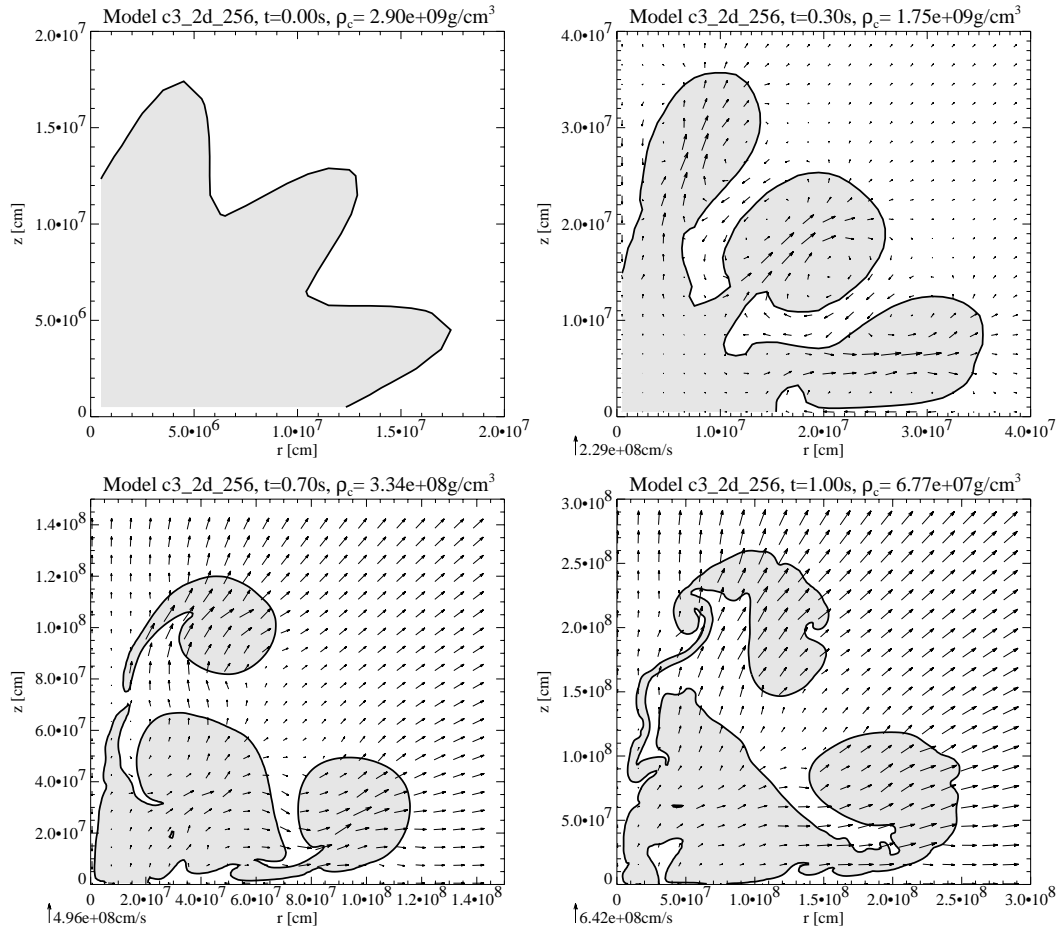


Fig. 2. Burning front geometry evolution of model c3_2d_256.

3.2 Comparison of 2D and 3D simulations

In order to investigate the fundamental differences between two- and three-dimensional simulations, a 2D and a 3D model with identical initial conditions and resolution was calculated. Figures 2 and 3 show snapshots of the flame geometry at various explosion stages; the energy evolution of both models is compared in figure 4.

It is evident that both simulations evolve nearly identically during the first few tenths of a second, as was expected. This is a strong hint that no errors were introduced into the code during the enhancement of the numerical models to three dimensions. At later times, however, the 3D calculation develops instabilities in the azimuthal direction, which could not form in 2D because of the assumed axial symmetry. As a consequence the total burning surface and the energy generation rate is increased, resulting in a higher overall energy release.

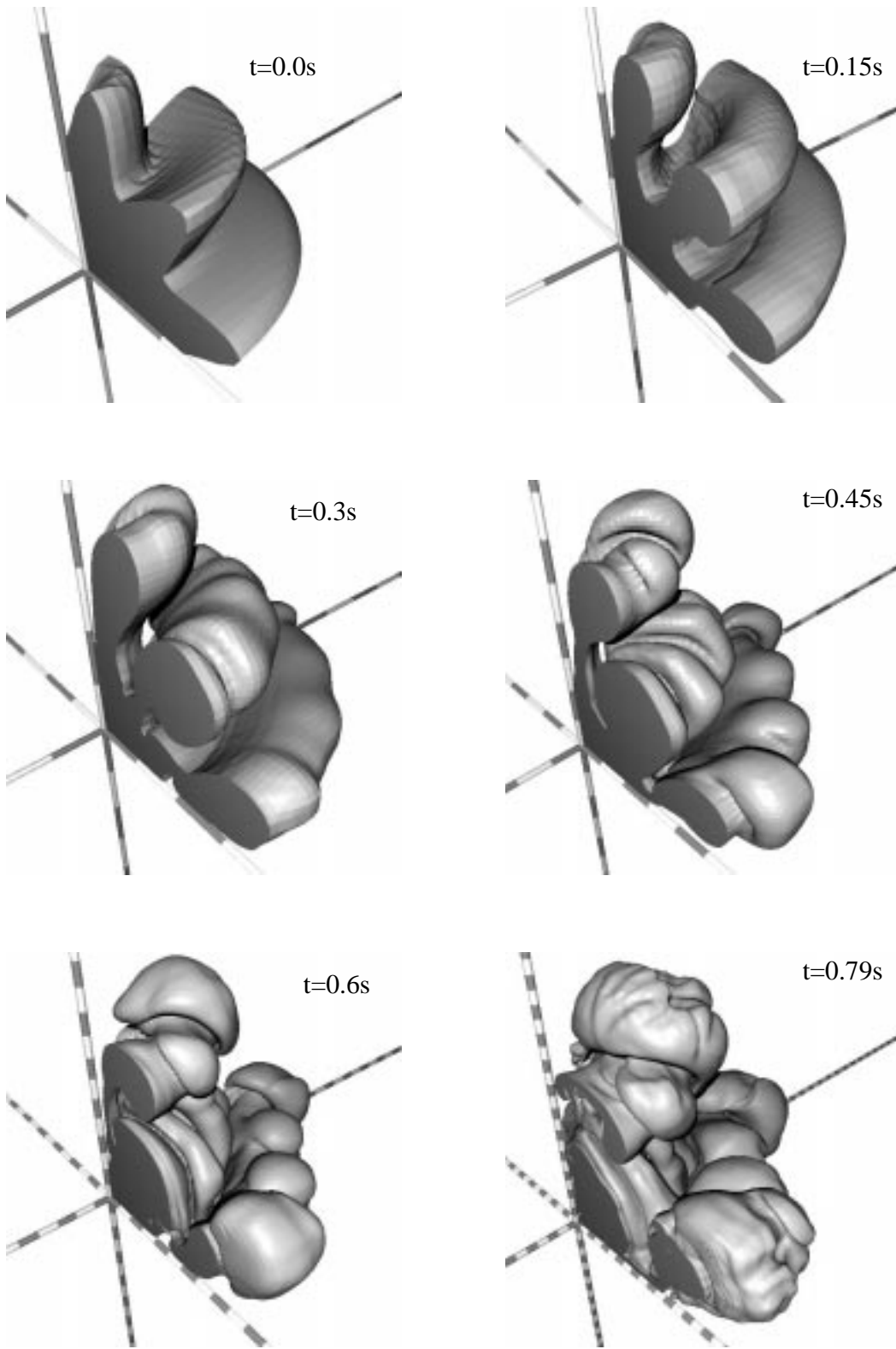


Fig. 3. Burning front geometry evolution of model c3_3d_256. One ring on the axes corresponds to 10^7 cm.

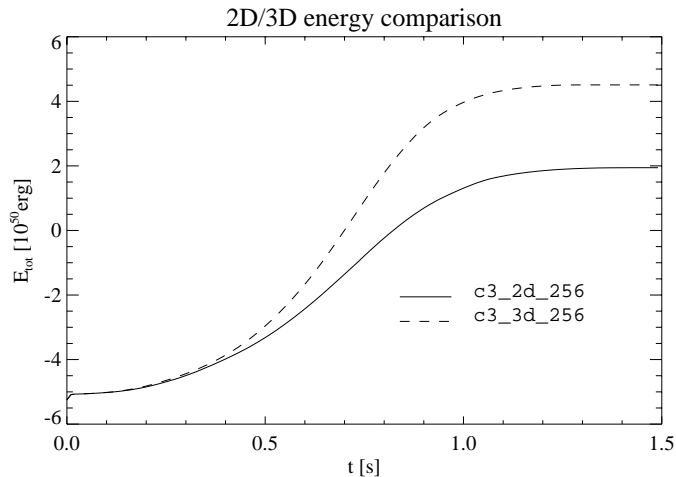


Fig. 4. Comparison of the energy evolution for identical initial conditions in two and three dimensions.

4 Conclusions

Table 1 lists the energy releases, as well as the masses of intermediate elements and nickel for the three-dimensional simulations performed up to date. These results (in contrast to the 2D models, which give too weak explosions) agree fairly well with the energies and nickel masses derived from observations (?). The results exhibit noticeable scatter for different initial conditions, i.e. the location of the flame at the beginning of the thermonuclear runaway seems to have an important influence on the supernova energetics. Since very little is known about this parameter, one has to investigate, as a next step, the secular pre-ignition evolution of the white dwarf, which finally determines the ignition conditions.

model name	$m_{\text{Mg}} [M_{\odot}]$	$m_{\text{Ni}} [M_{\odot}]$	$E_{\text{nuc}} [10^{50} \text{ erg}]$
c3_3d_256	0.177	0.526	9.76
b5_3d_256	0.180	0.506	9.47
b9_3d_512	0.190	0.616	11.26

Table 1

Overview over element production and energy release of the supernova simulations performed with the presented code. In contrast to the centrally ignited model c3_3d_256, models b5_3d_256 and b9_3d_512 were ignited in 5 resp. 9 spherical bubbles, which were randomly distributed in the vicinity of the center. In addition, model b9_3d_512 was calculated at higher resolution.

In addition to the good agreement with observations, it should be noted that our results also are quite similar to those obtained by ?), who approached the task of modeling the deflagration in a SN Ia in a quite different way. This could indicate that our understanding of all relevant physical processes is now

sufficient to devise correct numerical models and converge towards the real solutions.